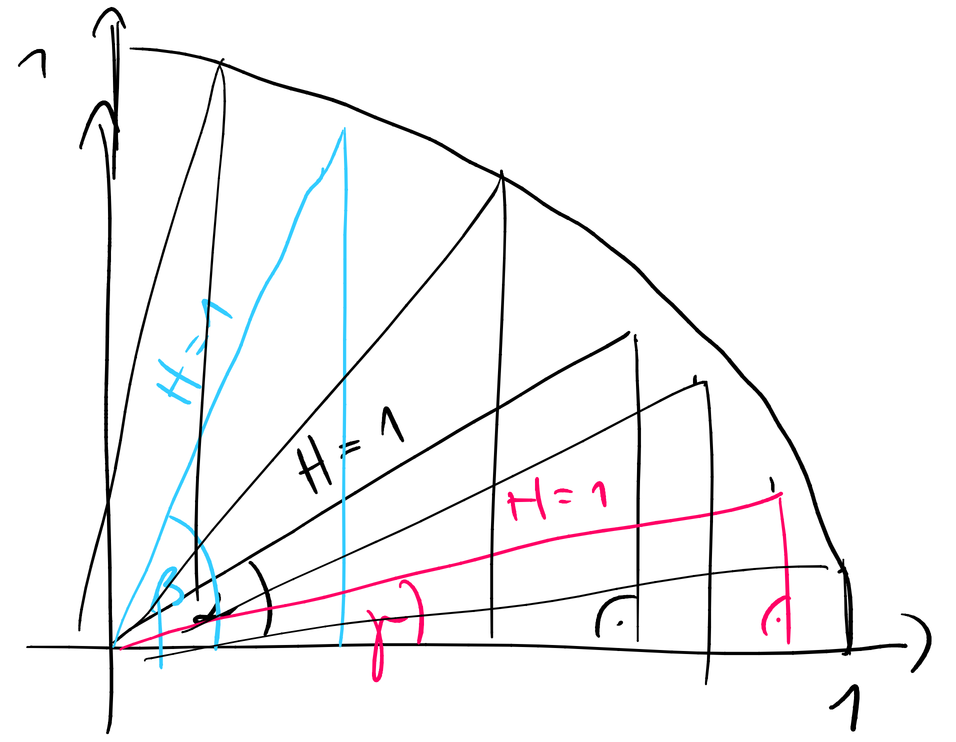
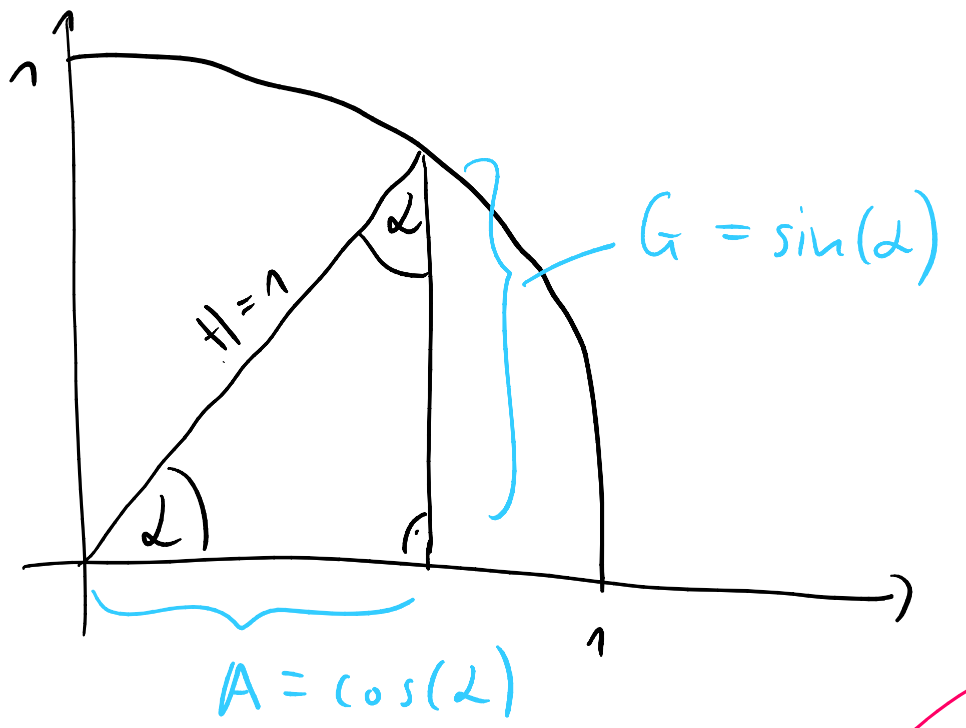
10.09.18 - Notizen

Montag, 10. September 2018 10:49

Was haben wir am Freitag gemacht?

- Dreiecke (kongruent, ähnlich)
- Strahlensätze
- Herleitung von Sinus, Kosinus, Tangens, Cotangens am Einheitskreis,
- Beträge





$\alpha = \frac{\pi}{4} = 45°$

$\sin\left(\frac{\pi}{4}\right) = G$

$\cos\left(\frac{\pi}{4}\right) = A$

$\Rightarrow \boxed{A = G}$, da $\triangle$ gleichschenklig ist.

Pythagoras: $\boxed{A^2 + G^2 = 1^2}$

(Allgemeine Formel)

$\Rightarrow \sin^2(\alpha) + \cos^2(\alpha) = 1$

$1 = A^2 + G^2 = G^2 + G^2 = 2G^2 = 2\sin^2\left(\frac{\pi}{4}\right) \quad | :2$

$\Leftrightarrow \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right) \quad | \sqrt{\ }$

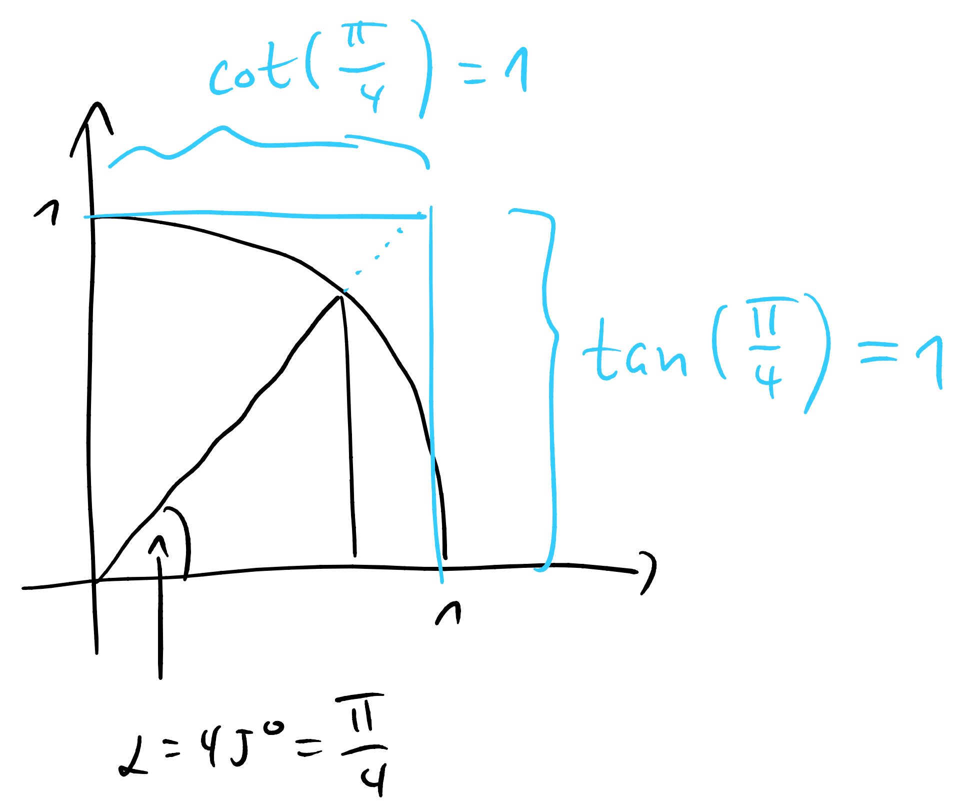
$$\Leftrightarrow \sqrt{\frac{1}{2}} = \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Da $A = G \Rightarrow \cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$$\tan\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\cancel{\frac{1}{\sqrt{2}}}}{\cancel{\frac{1}{\sqrt{2}}}} = 1$$

$$\cot\left(\frac{\pi}{4}\right) = \frac{\cos\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)} = \frac{\cancel{\frac{1}{\sqrt{2}}}}{\cancel{\frac{1}{\sqrt{2}}}} = 1$$



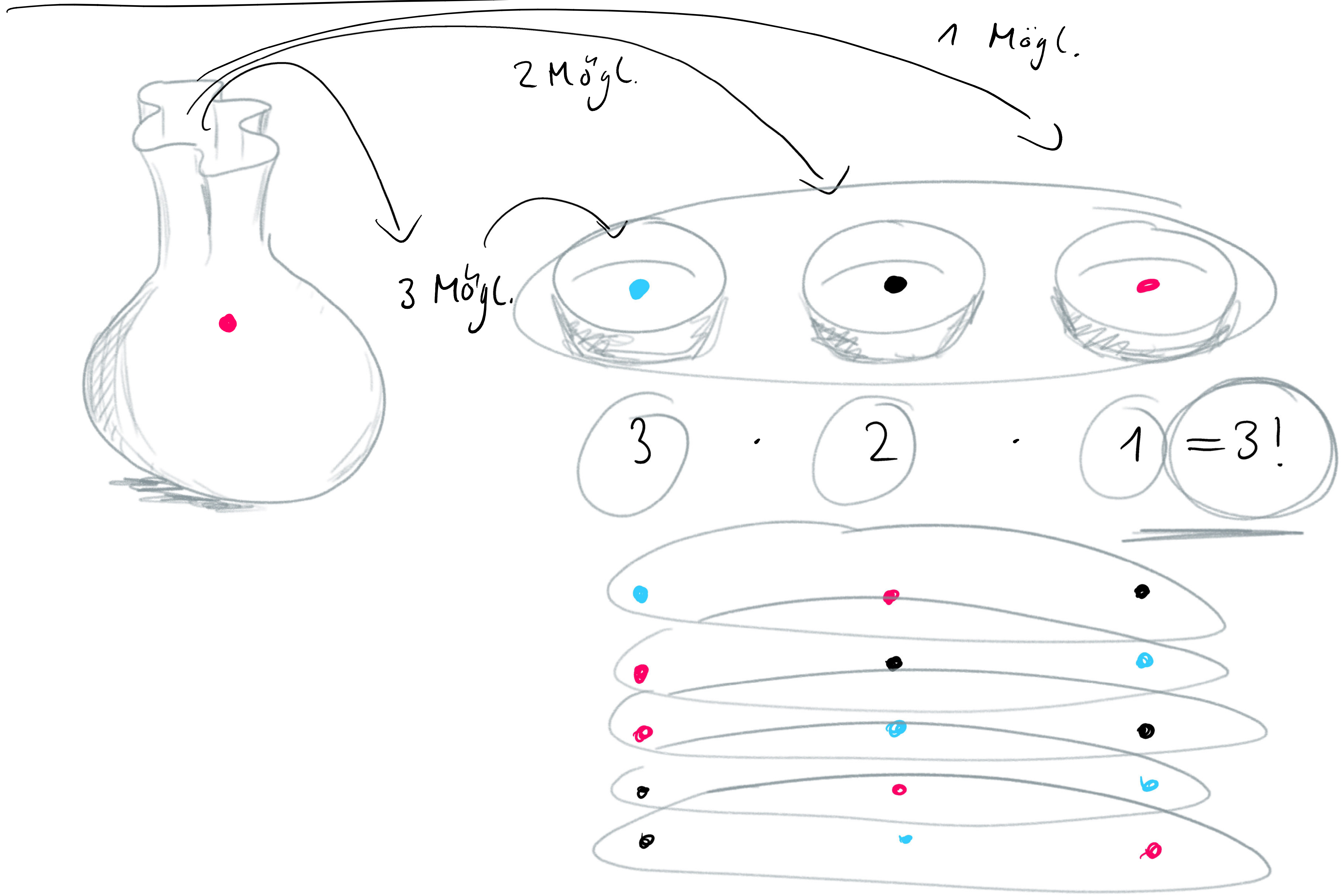
$$\sin(45° + 90°) = \sin\left(\frac{\pi}{4} + \frac{\pi}{2}\right) \neq \sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{\sqrt{2}} + 1 = \frac{1+\sqrt{2}}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{4} + \frac{2\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\boxed{\sin\left(\frac{\pi}{4}+\frac{\pi}{2}\right)=\sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{\pi}{2}\right)}$$

$$=\cancel{\frac{1}{\sqrt{2}}\cdot 0}+\frac{1}{\sqrt{2}}\cdot 1=\frac{1}{\sqrt{2}}$$



$$3\cdot 2\cdot 1=3!$$

$$\binom{3}{2}=\frac{3!}{(3-2)!\cdot 2!}=\frac{6}{1!\cdot 2!}=\frac{6}{1\cdot 2}=3$$

$$(n-k)!\cdot k!\neq((n-k)k)! \qquad 3\cdot(3!)\neq 9!$$

$$\binom{n}{n}=\frac{n!}{(n-n)!\,n!}=\frac{\cancel{n!}}{(0!)\,\cancel{n!}}=1 \qquad 0!\overset{\text{Definition}}{=}1$$

$$\binom{n}{n} = \frac{\quad}{(n-n)!\,n!} = \frac{\quad}{0!\,\cancel{n!}} = 1 \qquad 1 = 0!$$

$$3! = 1\cdot 2\cdot 3$$

$$n! = 1\cdot 2 \cdot \ldots \cdot n$$

$$0! \neq 1\cdot 0 = 0$$

$$\binom{n}{0} = \frac{n!}{(n-0)!\,0!} = \frac{n!}{n!\,0!}$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{n-k} = \frac{n!}{\underbrace{(n-(n-k))}!\,.\,(n-k)!} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{100}{98} = \binom{100}{100-98} = \binom{100}{2} = \frac{100!}{98!\cdot 2!} = 4950$$

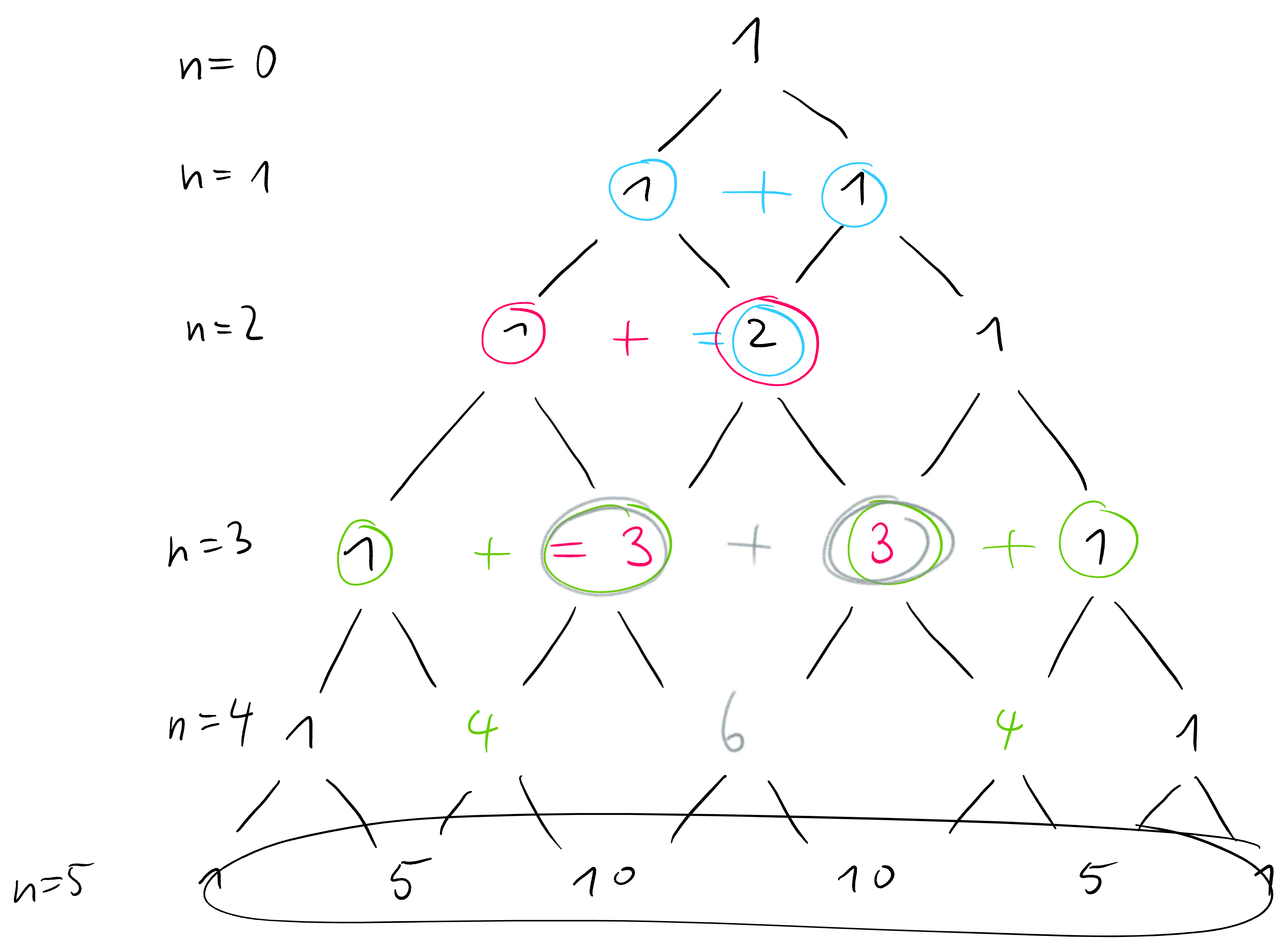
$$= \frac{\cancel{98!}\cdot 99 \cdot \cancel{100}^{50}}{\cancel{98!}\cdot\cancel{2}} = 99\cdot 50 = 4950$$

$$= (100-1)\cdot 50 = 5000 - 50$$

$$\binom{n}{k} + \binom{m}{l} \neq \binom{n+m}{k+l}$$

Binomialkoeff. ist <u>kein</u> Vektor

<u>Pascalsche Dreieck</u>



$$(a+b)^4 = (a+b)(a+b)(a+b)(a+b)$$

$$= (a+b)^2(a+b)^2$$

$$= (a^2+2ab+b^2)(a^2+2ab+b^2)$$

$$= ((a^2+2ab)+b^2)^2$$

$$= (a^2+2ab)^2 + 2(a^2+2ab)b^2 + (b^2)^2$$

$$= a^4 + 4a^3b + \underbrace{4a^2b^2 + 2a^2b^2}_{} + 4ab^3 + b^4$$

$$= 1\cdot a^4 \cdot b^0 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4\cdot a^0$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

$$(a+b)^5 = \underset{\binom{5}{0}}{1} \cdot a^5 \cdot b^0 + \underset{\binom{5}{1}}{5} \cdot a^4 \cdot b^1 + \underset{\binom{5}{2}}{10} a^3 b^2 + \underset{\binom{5}{3}}{10} a^2 b^3 + \underset{\binom{5}{4}}{5} \cdot a^1 b^4 \cdot \underset{\binom{5}{5}}{1} \cdot a^0 \cdot b^5$$

$$= 1\, a^5 b^0 + 5\, a^4 b^1 + 10\, a^3 b^2 + 10 a^2 b^3 + 5 a^1 b^4 + 1 a^0 b^5$$

$$(a+b)^5 \neq a^5 + b^5$$

$$(a-b)^3 = (a+(-b))^3 = a^3(-b)^0 + 3a^2(-b) + 3a(-b)^2 + a^0(-b)^3$$

$$= a^3 - 3a^2b + 3ab^2 - b^3$$

$$(-b)^3 = -b^3 \qquad (-b)^2 = b^2$$

$$\| $$

$$(-1)^3 b^3 = -b^3$$

$$(-1)^{n} = \begin{cases} 1, & n \text{ gerade} \\ -1, & n \text{ ungerade} \end{cases}$$

$$(-1)^n \cdot (-1)^n = \begin{Bmatrix} 1, & n \text{ gerade} \\ -1, & n \text{ ungerade} \end{Bmatrix} \cdot \begin{Bmatrix} 1, & n \text{ gerade} \\ -1, & n \text{ ungerade} \end{Bmatrix}$$

$$= \begin{Bmatrix} \overbrace{1 \cdot 1}^{=1}, & n \text{ gerade} \\ \underbrace{(-1)\cdot(-1)}_{=1}, & n \text{ ungerade} \end{Bmatrix} = 1$$

$$(-1)^n \cdot (-1)^n = (-1)^{n+n} = (-1)^{2n} = 1$$

gerade

$$= (-1)^{2n} = \left((-1)^2\right)^n$$

$$= (1)^n = 1^n = 1$$

$$49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 = \frac{49!}{43!} = \frac{49!}{(49-6)!}$$

Möglichkeit, 6 Zahlen mit beliebiger Reihenfolge zu ziehen, $\frac{49!}{(49-6)!\,6!} = \binom{49}{6}$