NR:

① $\mathbb{L}=\{\sqrt[10]{4}, -\sqrt[10]{4}\}$ ④ $\mathbb{L}=\{\sqrt[15]{3}\}$

② $\mathbb{L}=\{\}=\emptyset$ ⑤ $\mathbb{L}=\{0\}$

③ $\mathbb{L}=\{\sqrt[15]{-3}\}$ ⑥ $\mathbb{L}=\{\}=\emptyset$

$$\boxed{x^n = a}$$

$$6p - \frac{1}{2}(2p-3) = 3(1-p) - \frac{7}{6}(p+2) \quad | \text{ Distributivgesetz}$$

$$\Leftrightarrow 6p - \frac{1}{\cancel{2}} \cdot \cancel{2}p + \frac{3}{2} = 3 - 3p - \frac{7}{6}p - \frac{7}{3} \quad | \cdot 6 \text{ (kgV)}$$

$=2 \cdot 3$

$$\Leftrightarrow 36p - 6p + 9 = 18 - 18p - 7p - 14$$

$$\Leftrightarrow 30p + 9 = 4 - 25p \quad | +25p, -9$$

$$\Leftrightarrow 55p = -5 \quad | :55$$

$$\Leftrightarrow p = -\frac{5}{55} = -\frac{1}{11}$$

$\mathbb{L} = \left\{ -\frac{1}{11} \right\}$

$$\frac{55}{6} p = -\frac{5}{6} \qquad \Big| : \frac{55}{6}$$

?

$$p = -\frac{5}{6} \cdot \frac{1}{\frac{55}{6}} \ ?$$

$$p = -\frac{5}{6} : \frac{55}{6} \ ?$$

Sei $x \in \mathbb{R} \setminus \{0, -3\}$.

$$A(x): \boxed{\frac{2x^2 + 5x - 9}{x(x+3)} = \frac{2}{x+3} + 1} \qquad \Big| \cdot x(x+3) \quad (\text{kgV})$$

$$\Leftrightarrow \cancel{\frac{2x^2+5x-9}{x^2+3x} = \frac{2}{x+3} + 1}$$

$$\Leftrightarrow 2x^2+5x-9 = 2x + x\cdot(x+3) \quad \Big|\ \text{Distributiv-gesetz}$$

$$\Leftrightarrow 2x^2+5x-9 = 2x + x^2+3x$$

$$\Leftrightarrow 2x^2+5x-9 = 5x + x^2 \quad \Big|\ -5x-x^2$$

$$\Leftrightarrow x^2 - 9 = 0 \quad \Big|\ +9$$

$$\Leftrightarrow x^2 = 9$$

$$\Leftrightarrow x = 3 \ \vee\ \cancel{x = -3}$$

$$\mathbb{L} = \{3\} = \{3, \cancel{-3}\}$$

$$\mathbb{L} = \{ x \in \mathbb{R} \setminus \{0,-3\} : A(x) \text{ wahr ist} \} = \{3\}$$

$$x^2 + 6x + 1 = 0$$

Ziel: $x = \ldots$

quadratische Ergänzung:
(Idee: Benutze binomische Formel)

$$x^2 + 6x + 1 \stackrel{?}{=} (x+a)^2 + b = \underline{x^2 + 2ax + a^2 + b}$$

$$= (x+3)^2 + b = x^2 + 6x + 9 + b$$

$$= (x+3)^2 - 8 = x^2 + 6x + \underbrace{9 - 8}_{=1}$$

$$x^2 + 6x + 1 \overset{!}{=} (x+a)^2 + b$$

Wähle a, b geschickt (wie oben). $a = \frac{6}{2} = 3$

$$x^2 + 5x + 1 = \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 1$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{4}{4}$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{21}{4} = 0 \quad \Big| + \frac{21}{4}$$

$$\Leftrightarrow \left(x + \frac{5}{2}\right)^2 = \frac{21}{4}$$

$$\iff x + \frac{5}{2} = \sqrt{\frac{21}{4}} \quad \vee \quad x + \frac{5}{2} = -\sqrt{\frac{21}{4}}$$

$$\iff x = \frac{\sqrt{21}}{2} - \frac{5}{2} \quad \vee \quad x = -\frac{\sqrt{21}}{2} - \frac{5}{2}$$