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Im Rahmen der

AG Komplexe Analysis

laden wir zu folgender Vortragsreihe ein:

Rescaling Methods in Complex Analysis

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Die Vorträge finden statt in der Zeit **06.06.2023 bis 29.06.2023** in den Räumen G.15.25 und G.10.03 (Hörsaal 08) der Bergischen Universität Wuppertal.

Abstract:
The Rescaling Method in Complex Analysis and Geometry was introduced by S. I. Pinchuk via the terminology *stretching coordinates* around 1979 and by S. Frankel (another version) via the term *affine blow-up* around 1988, and further studied and developed by many excellent authors. Over time, it has been called by the terminology *scaling method*. It is actually the method of the infinite sequence of holomorphic magnification that rescales the domains in $\mathbb{C}^n$ (hence the 'new' term *rescaling method*). This simple idea has produced many results; indeed, learning this theory is the aim of this course.

Lecture 1 & 2 (Tuesday, 06.06., 16:00 c.t., G.10.03)

1. Rescaling method in $\mathbb{C}$.
   The aim of this first lecture is to get acquainted with the idea of the Rescaling method, especially for domains with non-compact automorphism group in the complex plane $\mathbb{C}$. Here, we compare a classical theorem and its proof method [16] with the 'rescaling method'.

2. Wong-Rosay theorem, I.
   The subject of this lecture is the Wong-Rosay theorem. We shall discuss the original proof by Wong [20] and the improvement by Rosay [19]. Here we shall learn how to use the arguments by H. Wu [21] using holomorphic invariants such as the quotient of the Carathéodory volume by the Kobayashi volume. Then we shall learn the methods by Wong and Rosay including the Localization methods by the noncompact sequence of automorphisms, the boundary behavior of holomorphic invariants, etc.

Lecture 3 & 4 (Tuesday, 13.06., 16:00 c.t., G.10.03)

3. Wong-Rosay theorem, II.
We shall learn the "first" version of the Rescaling method following Pinchuk [18]; it proves Wong's theorem in particular.

4. Pinchuk's rescaling method.
Pinchuk's "stretching coordinate method" will be discussed. This is really the rescaling method (at least the first version, if not the best), and it consists of localization, centering, and stretching sequences. We shall analyze this method to form the fundamentals for further study of Complex Analysis and Geometry.

Lecture 5 & 6 (Thursday, 15.06., 16:00 c.t., G.15.25)

5. Rescaling method by S. Frankel.
Another rescaling method was introduced by S. Frankel [3]. This applies to only the Kobayashi hyperbolic convex domains in $\mathbb{C}^n$, but has unique merits. In particular, this method produces a 1-parameter subgroup of the automorphism group of the domain from a divergent (non-compact) sequence of automorphisms.

6. Modified rescaling sequences.
One wonders at this stage whether the two rescaling methods are really different. We shall make a reasonable adjustment to the rescaling methods and discuss their equivalence. Cf. [10], [6].

Lecture 7 & 8 (Tuesday, 20.06., 16:00 c.t., G.10.03)

7. Bedford-Pinchuk theorem.
Wong's theorem was generalized by Bedford and Pinchuk [1]. It gives a characterization of a bounded pseudoconvex domain with a real analytic boundary possessing a non-compact automorphism group. The main method of the proof is the rescaling method by Pinchuk. We shall discuss the outline of the proof in as much detail as possible for a 1-hour lecture.

8. Analytic polyhedra with non-compact automorphism group.
Rescaling methods continue to work for the case of Levi flat boundary. We shall discuss the analysis of convex analytic polyhedra with non-compact automorphism group by Kim [8], [11], [23].

Lecture 9 & 10 (Thursday, 22.06., 16:00 c.t., G.15.25)

9. Dynamics and Greene-Krantz conjecture.
The rescaling method is a version of Complex Dynamics. So we discuss the rescaling method by a sequence formed by iteration of holomorphic mappings, by [9]; [13].

10. Rescaling for CR submanifolds.
Since the rescaling method actually deals with the rescaling of the boundary (which was the case by Pinchuk's initial application of his rescaling method), Kim/Schmalz used the method in characterizing the quadric by its CR automorphisms [12]. This will be discussed in detail.

Lecture 11 & 12 (Tuesday, 27.06., 16:00 c.t., G.10.03)

11. On the Semicontinuity theorem for the automorphism group of domains.
One of the natural problems with a long research history is the semicontinuity problem of Lie groups acting on (compact) manifolds. This problem was studied by Greene and Krantz [5]. We shall discuss the original proof, which is more fitting to the Lie theory and Riemannian Geometry.

12. Semicontinuity theorem by rescaling methods.
The rescaling method works for this problem as well. In fact, it solves a new case as well [4]. We shall discuss the proof method in detail. Then we shall pose some related open problems.

Lecture 13 & 14 (Thursday, 29.06., 16:00 c.t., G.15.25)

13. Asymptotic behavior of holomorphic invariants.
The boundary behavior problem for holomorphic invariants had been generally regarded as a much more difficult problem than the problem of their interior stability. But the rescaling method makes it possible to use the latter to solve the former! We shall study this method initiated by Kim and J. Yu [14]. There are also other applications.

14. Squeezing function, rescaling methods and uniform squeezing property for bounded convex domains.
Liu-Sun-Yau papers (J. Diff. Geom. 2005) introduced the concept of Holomorphic homogeneous manifold in studying various equivalent problems between Riemannian metrics. Then S. K. Yeung [22] used it to obtain applications to complex geometry. Deng-Guan-Zhang [2] settled the terms "Uniform squeezing property" and "squeezing functions". Then J. E. Fornaess posed a question of whether some ball-like asymptotic behavior of squeezing function implies strong pseudoconvexity. We shall present the circle of results of various types related to these concepts. Some use the rescaling method explicitly. Cf. [7]. Rescaling also solves the question of whether (strongly) convex domains are uniformly squeezing (=holomorphic homogeneous regular). We shall discuss how these problems are solved [15, 22].

Alle Interessenten sind herzlich eingeladen!

gez. Prof. N. Shcherbina

Literatur

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