On multivariate Polya's inequality for Hankel determinants

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Let K be a compact set in $\mathbb{C}$ and $d(K)$ its transfinite diameter. Polya proved (1929) that for every function f analytic in $\overline{\mathbb{C}} \setminus K$ with the Taylor expansion

$$f(z) = \sum_{k=0}^{\infty} \frac{a_k}{z^{k+1}}$$

the following inequality holds

$$D(f) := \limsup_{s \to \infty} |H_s(f)|^{1/s^2} \leq d(K),$$

where $H_s(f) = \det (a_{k+l})_{k,l=0}^{s-1}$, $s \in \mathbb{N}$, is the sequence of Hankel determinants. Goluzin gave (1946) some sufficient conditions on the compactum K under which this inequality is sharp, that is $d(K) = \sup \{D(f)\}$, where supremum is taken over all functions f analytic in $\overline{\mathbb{C}} \setminus K$.

We discuss multivariate analogs of the transfinite diameter and Polya's inequality. Some multivariate sharpness results will be considered, which seems to be new even for the case of one variable (these results are joint with my Ph. D. student Gunyuz).